First, $3x+2z=3[3y+2u]\ldots(1)$;
Second, $2x+6v=8\{2[1-(x+y)]+6[1-(v+w)]\}\ldots(2)$;
Third, $2u+3w=5\{2[1-(z+u)]+3[1-(v+w)]\}\ldots(3)$;
Fourth, $x+2z+3v=2(y+2u+3w)\ldots(4)$;
Fifth, $x+z : y+u :: 11 : 5\ldots(5)$; and
Sixth, $6x+5z+2v : 13 :: 17 : 24\ldots(6)$.

By elimination, we have $x=1$, $y=0$, $1-(x+y)=0$, $z=\frac{3}{8}$, $u=\frac{5}{8}$, $1-(z+u)=0$, $v=\frac{2}{3}$, $w=\frac{5}{24}$, and $1-(v+w)=\frac{1}{8}$.

We interpret these results as follows: That the first ingot was pure gold; that the second ingot was 9-carat gold, and 15-carat silver; and that the third ingot was 16-carat gold, 5-carat silver, and 3-carat copper.

Also solved in the same manner by G. B. M. Zerr, S. A. Corey, L. E. Newcomb, and G. W. Greenwood.

274. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Find the limit of $\frac{3^2+1}{3^2-1}\cdot\frac{5^2+1}{5^2-1}\cdot\frac{7^2+1}{7^2-1}\cdot\frac{11^2+1}{11^2-1}\ldots$ where the squared numbers are the natural odd *primes* in order.

Solution by G. B. M. ZERR, Ph. D., Parsons, W. Va., and J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Putting the expression in the form

$$\frac{(1+1/3^2)(1+1/5^2)(1+1/7^2)\ldots}{(1-1/3^2)(1-1/5^2)(1-1/7^2)\ldots}=\frac{s}{s^1}$$

and remembering that $(1+1/2^2)s=\frac{15}{\pi^2}$ and $(1-1/2^2)s_1=\frac{6}{\pi^2}$, (pp. 133-134, Vol. V, No. 5), we have $s/s_1=1\frac{1}{2}$.

C. N. Schmall gives the following arithmetical solution of 269. When the boats first meet, combined distance traveled is equal to width of river; when they meet for the second time the distance traveled is equal to three times the width of river and that each boat has gone three times as far as when they first met. Hence one has gone 3×720 yards$=$ 2160 yards and has made one trip and 400 yards of the return trip· Hence, width of river $=$2160 yards—400 yards$=$1760 yards$=$1 mile.

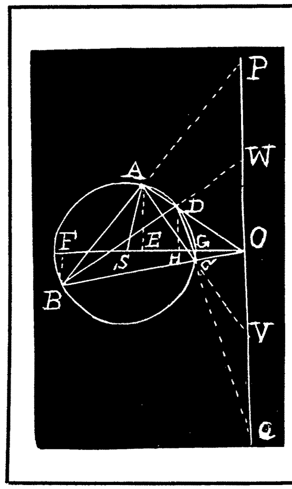
GEOMETRY.

302. Proposed by F. H. SAFFORD, Ph. D., University of Pennsylvania.

Through a given point within a circle draw any two chords, also a radius and a secant perpendicular to the radius. Let the extremities of the chords be taken as the vertices of a quadrilateral. Show that the sides of the quadrilateral, produced when necessary, cut the secant in points equidistant, in pairs, from the given point. [A proof by Euclidean geometry is preferred, as the problem was originally given to a high school class.] Must the given points be within the circle?

Solution by the PROPOSER.

The problem holds for the complete quadrilateral and for the given point taken inside or outside of the circle. Of the three pairs of intercepts on the secant, one is null, corresponding to the given point. The following proof is for the case of the given point outside of the circle and the intercepts by the *diagonals* upon the secant. The method is applicable to all cases.



In the diagram, O is the given point, A, B, C, D are the vertices of the quadrilateral, and E, F, G, H are the projections of the respective vertices upon the diameter determined by O and the center of the circle at S. Let a be the radius of the circle and let SO be b. To prove that the diagonals AC and BD, produced, cut off equal distances OV and OW on the perpendicular to SO at O. From the similar triangles OAC and OBD,

$$\frac{\triangle OAC}{\triangle OBD}=\frac{OA^2}{OB^2}=\frac{OC^2}{OD^2}\ldots(1).$$

Since $\triangle OAC=\triangle OAV-\triangle OCV$, $\triangle OBD=\triangle OBW-\triangle ODW$, and using OV and OW as bases, it follows that

$$\frac{\triangle OAC}{\triangle OBD}=\frac{OV(OE-OG)}{OW(OF-OH)}\ldots(2).$$

From (1) and (2),

$$\frac{OV}{OW}=\frac{OA^2}{OB^2}\cdot\frac{OF-OH}{OE-OG}\ldots(3).$$

Noticing that OE and OG are the projections of OA and OC, the triangles SOA and SOC give, $a^2=b^2+OA^2-2b.OE$, $a^2=b^2+OC^2-2b.OG$.

$$\left.\begin{aligned}\therefore OE-OG&=\frac{OA^2-OC^2}{2b}\\ \text{Similarly, } OF-OH&=\frac{OB^2-OD^2}{2b}\end{aligned}\right\}\ldots(4).$$

From (3) and (4), using also the latter part of (1),

$$\frac{OV}{OW}=\frac{OA^2}{OB^2}\cdot\frac{OB^2-OD^2}{OA^2-OC^2}=\frac{OA^2.OB^2-OA^2.OD^2}{OB^2.OA^2-OB^2.OC^2}=1.$$

Also solved by C. N. Schmall, A H. Holmes, and L. E. Newcomb. Mr. Newcomb's demonstration was exhaustive, covering the cases when the point is within and without the circle. Our space is too limited to publish his demonstration.